Fibonacci Sequence Weighted SAR ADC as Golden Section Search of Unimodal Function

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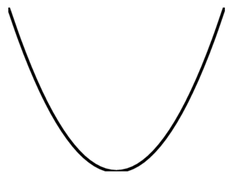
Introduction

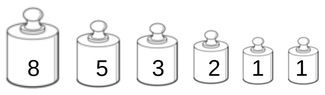
New Discovery

SAR ADC based on golden section search algorithm using a unimodal function

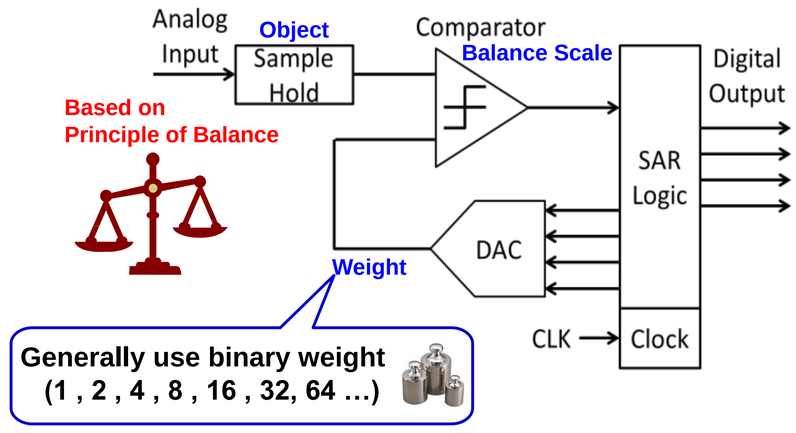


equivalent

Fibonacci sequence weighted SAR ADC



SAR ADC

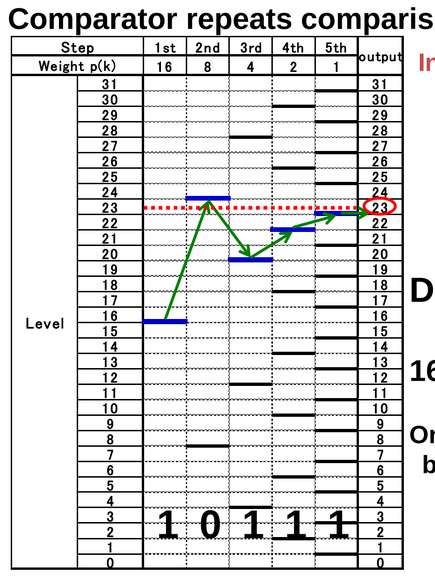


Fea

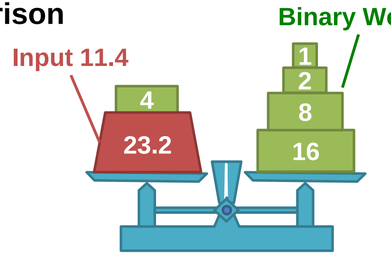
- High resolution • Medium sampling speed
- Small chip area • Low power

Binary Search SAR ADC Operation

Comparator repeats comparison



Binary Weight



Dout = 10111

16 + 8 – 4 + 2 + 1 + 0.5 – 0.5= 23

One-to-one map between Decimal and Binary codes

1 Misjudgment leads to incorrect output

Golden Section Search and Fibonacci Search Method

Fibonacci Sequence

Definition (n=0,1,2,3…)

$$F_0 = 0$$
$$F_1 = 1$$
$$F_{n+2} = F_n + F_{n+1}$$

Example of numbers(Fibonacci number)

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55…

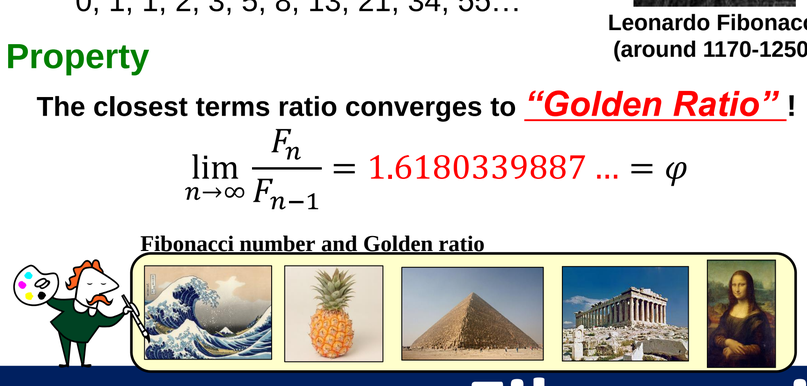
Leonardo Fibonacci (around 1170-1250)

Property

The closest terms ratio converges to *"Golden Ratio"* !

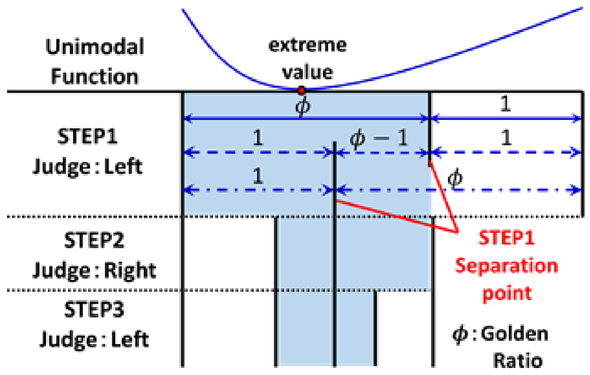
$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = 1.6180339887\ldots = \varphi$$

Fibonacci number and Golden ratio



Golden Section Search

- **Effective finding method of the extreme value of the unimodal function.**
- **Division ratio is golden ratio.**

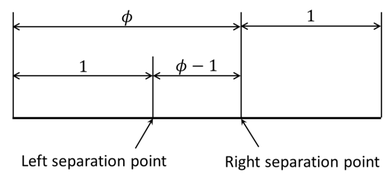


ADC treats only integers. ➡ **Difficult to realize**

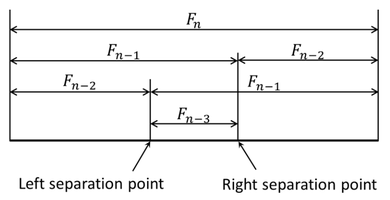
Fibonacci Search Method

Golden division method with only integers.

Golden Section Search (ϕ:Golden ratio)



Fibonacci search method (F_x:Fibonacci number)

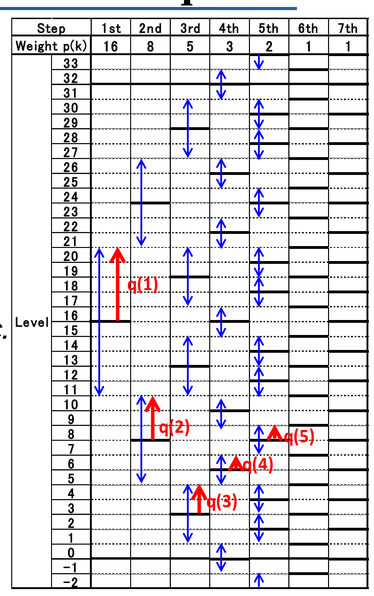


Fibonacci Sequence Weighted SAR ADC and Golden Section Search SAR ADC

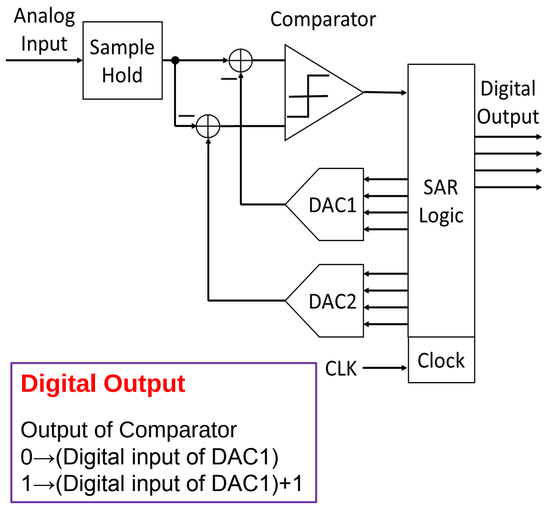
SAR ADC Using Fibonacci Sequence

Three properties are discovered !!

1. ***Correctable difference q(k) is always Fibonacci number F_{M-k-1}.***
2. ***q(k) is exactly in contact q(k+1) without overlap.***
3. ***Considering DAC output imperfection, it is the most fastest SAR ADC.***



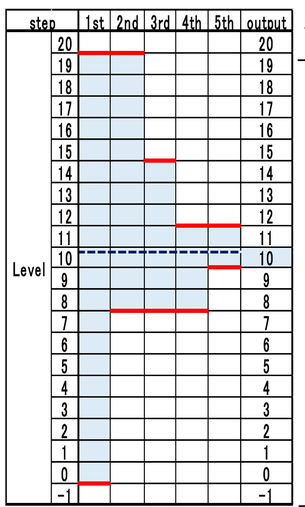
Golden Section Search SAR ADC



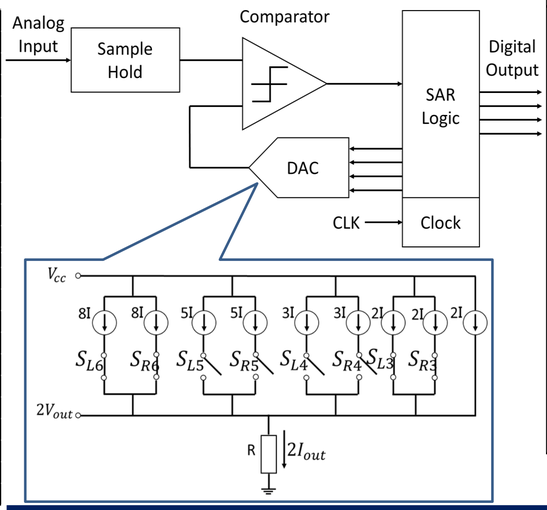
Digital Output

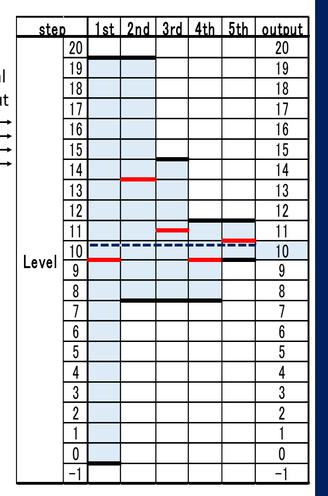
Output of Comparator
0→(Digital input of DAC1)
1→(Digital input of DAC1)+1

Solution chart when $V_{in} = 10.7$



Revised Golden Section Search SAR ADC





summary

New Theorem

Equivalency between

- SAR ADC using golden section search
- Fibonacci sequence weighted SAR ADC

Automotive Application

Proof by Numerical Formula

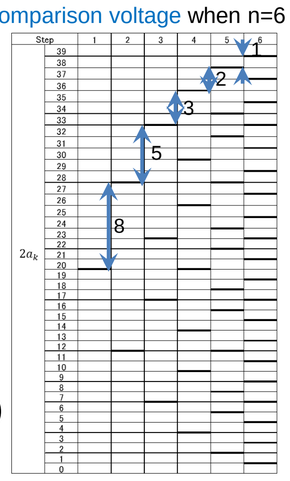
Fibonacci sequence

$$F_0 = 0$$
$$F_1 = 1$$
$$F_{n+2} = F_n + F_{n+1}$$

Comparison voltage

$$2a_k = (F_n + F_{n-1} + \cdots + F_1) + (\pm F_n \pm F_{n-1} \pm \cdots \pm F_1)$$

Comparison voltage when n=6

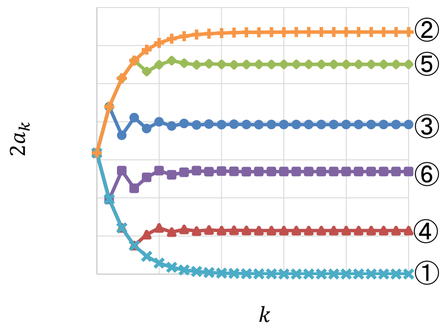


Difference between adjacent comparison voltages is Fibonacci sequence ➡ Matches Fibonacci sequence weighted SAR ADC

Simulation

Simulation result ※ $n = 26$

①When $V_{in} = 0$
②When $V_{in} = S_n = 317810$
③When $V_{in} = (1.2349/2)S_n = 196231.78$
④When $V_{in} = (0.35931/2)S_n = 57096.156$
⑤When $V_{in} = (1.7325/2)S_n = 275302.91$
⑥When $V_{in} = (0.8469/2)S_n = 134576.64$



All converged to the intended value

References

1. T. Ogawa, H. Kobayashi, Y. Takahashi, N. Takai, M. Hotta, H. San, T. Matsuura, A. Abe, K. Yagi, T. Mori, "SAR ADC Algorithm with Redundancy and Digital Error Correction", IEICE Trans. Fundamentals, vol. E93-A, no.2, pp. 415-423 (Feb. 2010).
2. Y. Kobayashi, S. Shibuya, T. Arafune, S. Sasaki, H. Kobayashi, "SAR ADC Design Using Golden Ratio Weight Algorithm" , International Symposium on Communication and Information Technologies, Nara, Japan (Oct. 2015)
3. Y. Kobayashi, H. Kobayashi, "Redundant SAR ADC Algorithm Based on Fibonacci Sequence", Advanced Micro-Device Engineering VI, Key Engineering Materials, pp. 117-126 (2016).